THE EFFECT OF MATHEMATICAL MODELS IN THEOREM IMPLEMENTATIONS ON PRE-SERVICE PRIMARY SCHOOL MATHEMATICS TEACHERS’ ACADEMIC ACHIEVEMENT

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This research study investigated the change in students’ academic achievement with an achievement test designed for the implementations of theorem proofs carried out with the help of mathematical models. For that purpose, the study was carried out with 45 second-grade students studying in the Department of Primary Education Mathematics Teaching in 2014-2015 academic year. One-group pre-test post-test model, a weak experimental design, was used in the study. The data of the study were carried via Mathematical Knowledge Test. Descriptive analysis and t-test were used for the data analysis. The analysis results of the data revealed that proofs of theorem carried out with the help of mathematical models increased students’ academic achievement. Researchers are suggested that this study should be implemented not only with theorem proofs but also with the visualisation of any concept that is difficult to understand and with problem solutions. In addition, constructing proofs for undergraduate studies is an inseparable part of mathematics education. However, students have difficulty in understanding this subject. Thus, doing proof activities with mathematical models course can be involved in undergraduate studies and students can understand proofs much better. In addition to this, students and teachers’ use of mathematical models can be developed. It is also suggested that sample activities in which models are used should be included in curricula at each level for students to understand models much better and use them in daily life.
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Keywords: Mathematical Proof, Theorem, Mathematical Theorem, Primary School Mathematics Students

Abstract


Anahtar Kelimeler: Matematiksel İspat, Teorem, Matematiksel Model, İlköğretim Matematik Öğrencileri

INTRODUCTION

Mathematical reasoning is a high level activity and one way to determine mathematical reasoning is to view how mathematicians have proven their theorems. Proof is an important tool used in mathematical reasoning and according to Selden and Selden (2003), they are propositions used to show the accuracy of the theorems. Bell (1976) regards proof as a process which is realized as a result of some steps and he emphasizes that categories of justification (verifying that a statement is true), explanation (proving why a statement is true), and systematization (organizing statements, axioms, and theorems in a deductive system) should exist in this process. Similarly, Baki (2008) regarded mathematical proofs as a process and he identified this process as justification, explanation, and abstraction.

Because axioms, definitions, theorems, and proofs of theorems are indispensable for mathematics and they compose the foundation of mathematics, mathematical statements have to be proven (Heinze, & Reiss, 2003). When students prove the mathematical statements and formulas in schools, they learn that it is not only enough to know the latest form of the formulas and the results obtained must be explained with reasons (Güven, Çelik & Karataş, 2005). When the research studies carried out by many mathematicians and educators on the importance of proof are examined, proof is described as a very important and an indispensable part of mathematics, the foundation of mathematics, and the purpose of advanced mathematics (Heinze & Reiss, 2003; Weber, 2001). When the reasons for regarding mathematical proof that much is considered, making contributions to the students and having students acquire mathematical skills come into prominence. It is seen in these studies that mathematics and proof fit together, proof makes a very important contribution to the formation of mathematical knowledge, and it develops students’ mathematical reasoning skills (Gökşur et al., 2014).

Proof is considered to be central to the
discipline of mathematics and mathematics differs from the other disciplines because of this characteristic (Ünveren, 2010). Especially, mathematical proof is a subject matter which is emphasized in higher education. When students’ undergraduate programs are examined, students are expected to acquire high level of proving skills in course contents. Thanks to the major area courses students take in higher education, they are equipped with the subject of doing mathematical proof while being trained. Especially, students are informed about doing proof and proof techniques in many courses such as Abstract Mathematics, Analysis and Algebra. Although mathematical proof is very important and it is emphasized a lot in undergraduate studies, students who study advanced mathematics at university have difficulties in constructing proof (Weber, 2004). The reasons why students have difficulties in constructing proofs are that students have lack of knowledge about proof, they did not know definitions and where to use them, they could not understand the nature of proof and proof techniques and strategies, they were unable to use mathematical language, and they had lack of confidence (Anapa & Şamkar, 2010; Baker & Campbell, 2004; Edwards & Ward, 2004; Moore, 1994; Moralı et al., 2006; Weber, 2006).

Such structures like explaining why a statement is true or wrong and selecting and using proof types with different ways of reasoning are important while constructing proofs (Baki, 2008). Although proof is very important for mathematics and mathematical reasoning, it is considered as a difficult topic for students to understand. One of the reasons for this is that mathematics has an abstract structure. Another reason why students have negative attitudes towards doing proof is that they are unable to comprehend what is tried to be explained in the statement of the given theorem. Teaching students a concept, a statement, a phenomenon, or a theorem directly in mathematics courses causes difficulties with understanding and internalizing. However, if abstract mathematics concepts are taught by being concretized during instruction, this difficulty can be reduced or eliminated. At this point, it is considered that proving theorems with mathematical models can change students’ views on doing proofs.

In mathematics education, many concepts, knowledge, or statements need to be supported with different application fields and methods and strategies so that they can become more meaningful for students. Considering from this perspective, mathematical models can make important contributions to the learning and internalizing of a concept, phenomenon, or statement. According to Doruk (2010) the model is defined as conceptual structures existing in mind to interpret and understand these complex systems and structures and consisting external representations of these structures as a whole. Olkun and Uçar (2007) state that the model of mathematical concept is a picture, drawing, symbol or a concrete tool which shows the relationship which this concept contains within itself. Harrison (2001) organizes the reasons for the use of the models in learning environment in that way:

- **Simplification:** Because it offers an opportunity to visualize complex abstract concepts, objects and processes, it facilitates much easier perception with abstract concepts which are difficult to understand. The subjects which are concretized find a place much more quickly in a student’s mind. Because the subject matter becomes easy for the student, learning time shortens and more time is allocated to do exercises.

- **Exaggeration:** Models exaggerate the fundamental features of the subject or the process and they draw attention to the key characteristics of opinions. Particularly, if the model removes the unnecessary details and drawings, learning becomes more effective.
Familiarity: Thanks to animations and simulations, models come in many varieties and forms of simple objects. The models which are composed of familiar objects promote students’ comprehension more.

Accessibility: Students can reach models whenever they want and this facilitates repetition or individual work.

According to Ünveren (2010), a proof can be explained or interpreted via a model which is constructed through reaching generalizations and rules derived from special cases and examples in terms of mathematics and thus students can learn and comprehend mathematical proofs which are done via using a case model which corresponds to their lives much better than the proofs that are constructed by formal methods. According to Fischbein (1997) because of the formal language of the proof and different structure of the model language used in teaching, the model situation becomes an analogy for the individuals to remember the proof. Thus, the individuals can construct the model of any theorem as they understand and they feel as if they prove the theorem with the model.

When the research studies which investigated the effect of using mathematical models on academic achievement were explored, it was found that Çiltaş and Işık (2012) stated that modelling had affected academic achievement positively, Moslev and Jenaabadi (2015) found that using modelling activities had an important effect on students’ academic achievement and Yıldırım and Işık (2015) remarked that instruction carried out with modelling activities included in the curriculum was more effective in increasing academic achievement when compared to the curriculum which did not involve modelling activities. Therefore, modelling activities designed at different grade levels will not only develop students’ academic achievement in mathematics, modelling skills, and creativity but also they offer opportunities to train students who can produce different ways of solutions to the problems they encounter, have analytical thinking skills, and have qualities like reasoning and association (Çiltaş, 2015).

The research conducted on mathematical proof (Baker & Campbell, 2004; Blum, 1998; Ünveren, 2010; Güler, & Dikici, 2010; Güven et al., 2005; Weber, 2001; Yıldırım & Işık, 2015) stress that proof constitutes an important place in mathematics and the truth of a phenomenon, a concept, or a theorem cannot be verified without a proof. Although proof is very important, it is also revealed by the research that students have difficulties while doing proof and the reasons for the difficulties encountered by the students become another research topic. One of the most important factors which will promote understanding and internalization of proof is teachers’ teaching methods. In this respect, models can offer teachers and students opportunities to express themselves much more freely and calmly when they have difficulties in expressing their abstract opinions. Concretizing mathematical models and abstract theorem proofs, students can understand the proof much more easily because while actualizing proof, it is expected that a hypothesis is certainly taken from somewhere. For that purpose, if this hypothesis is chosen from students’ lives, students can have an instructional experience as if they construct the proof themselves for the first time. Thus, they can reconstruct the justifications and conceptual verifications inherent in the proof like a mathematician (Ünveren, 2010). It can be stated that using real-life situations which are taken from students’ lives as examples and which are away from formal structures will offer more effective, more permanent, more meaningful learning environments with such proofs. However, when literature is examined, there are not many studies carried out about using models in constructing proofs. Considering this view and the fact that mathematical models will help students to prove the theorems, this research study explored the effects of proving a theorem’s statement via mathematical models on students’ academic achieve-
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METHOD

The Research Model

One group pre-test-post-test model, a weak experimental design, was used in this study. In one group pre-test-post-test model, an independent variable is applied on a single group and measured before and after the treatment (Blum, 1998). Because the purpose of this study was to examine the effectiveness of the method implemented, weak experimental method, one of the quantitative research approaches, was considered as the plan of the research study.

The Study Group

The participants of the study are composed of 45 students in their second year of studies in the Department of Primary School Mathematics Teaching in 2014-2015 academic year. Purposeful sampling, one of the non-probability sampling techniques, was used to select the study group. Purposeful sampling is used for the identification and selection of information rich-cases for in-depth analysis (Büyüköztürk, vd., 2012). As the activities used in this study were taken from Analysis-I course, the students who were taking or took Analysis-I were considered when the participants studying in the second grade were chosen. Thus, the study group was chosen according to criterion sampling, one of the purposeful sampling methods. Criterion sampling involves selecting cases that meet some predetermined criterion of importance like people, objects, or conditions (Büyüköztürk, vd., 2012).

Data Collection Tools and Analysis

Mathematical Knowledge Test used in the study consisted of four open-ended questions in the pre-test and five open-ended questions in the post-test because the purpose of the test was to examine the effects of theorem implementations carried out with mathematical models on students’ academic achievement. During the design of the mathematical knowledge test by the researcher and considering the theorems, source books and literature were utilised (Akdeniz, Ünlü, & Dönmöz, 2006; Balci, 2008) and it was developed by taking the experts’ opinions. The participants were given a one hour lesson (50 minutes) to answer mathematical knowledge test.

Students’ responses were evaluated over 100 points so that the questions in the pre-test and post-test could have equal points and the data used to find out the change in students’ academic achievement were analysed using SPSS software package. Skewness was used in this study to view whether data are normally distributed or not. Skewness is divided by standard error of skewness and according to the results obtained, the distribution is normal for the values between +1.96 and -1.96, but the values above 1.96 or below -1.96 are considered normal up to the values in the range of +3 and -3 at 0.05 significance level (Kalaycı, 2010). Table 1 presents SPSS results of pre-test and post-test.

<table>
<thead>
<tr>
<th>Table 1. Pre-test and Post-test –SPSS results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-Test</td>
</tr>
<tr>
<td>N Valid</td>
</tr>
<tr>
<td>N Missing</td>
</tr>
<tr>
<td>Mean</td>
</tr>
<tr>
<td>Median</td>
</tr>
<tr>
<td>Mode</td>
</tr>
<tr>
<td>Std. Deviation</td>
</tr>
</tbody>
</table>
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When the results in Table 3.1 are examined, skewness/standard error of skewness equals to 0.772/0.354 = 2.18 > 1.96 and it is not normal in the pre-test. But according to (Kalaycı, 2010), because it was smaller than 3, it was considered normal in pre-test. In the post-test, because skewness/standard error of skewness equals to -0.038/0.354 = -0.107 > 1.96, the post-test was normally distributed. As the data in the study were normally distributed, t-test, one of the parametric tests, was used. Dependent t-test is a type of t-test which is used to compare the means and the analyses are performed on the same population (Kalaycı, 2010). Considering the results of the dependent t-test, effect size was also tested in terms of significance level. Effect size is a statistical value which shows the standard deviation from the expectations complemented in the accepted hypothesis by the results obtained from the samples (Field, 2009). According to Cohen (1992), if effect size is .10, the degree of effect is low, if it is .30, the degree of effect is medium and if it is .50 and above higher values, the degree of level is high (as cited in Field, 2009).

**Implementation**

Before the implementation of the study, piloting was carried out. It is very important to perform a pilot study because data collection tools are finalized and validity and reliability of the study were obtained. The piloting was administered with 44 students studying in their second year during the 2nd term of 2013-2014 academic year. Theorem proofs were done as activities. Then, expert opinions were taken, necessary changes were made and data collection tools were finalized.

The research study was carried out with 45 students in their second year at the university in 2014-2015 academic year and it was carried out between the 10th and 14th weeks of the fall term (total five weeks). The implementations lasted totally five weeks between 21 November and 26 December, 2014. In the first week, the knowledge test was administered as a pre-test. Then, activities were done for three weeks. There were total six activities and theorems were proved with each activity. Theorems were chosen from the course content of Analysis-I which was taught by using classical methods. The activities included Fermat, Rolle, Bolzano, and Intermediate Value for Integral and Sandwich (Squeeze) Theorem, respectively. Intermediate Value and the Sandwich Theorem were proved. One lesson hour (50 minutes) was allocated to each activity. At the beginning of the lesson, the activity worksheets were copied and handed out to the students and they were asked to prove the theorem. The students were given the first 25 minutes of the course for that stage. The researcher guided the students and all of the students finished the activity. At the end of this process, the researcher reminded the students the mathematical concepts included in the statement of the theorem. Before starting the theorem proof, the statement of the theorem was read and the model related to the theorem was constructed. Graph drawings were used as a model in the theorems. Then, the proof was done with the help of the graph drawn. It was ensured that students understood the proof in each stage of
the activity. At the end of the activities, related examples to the theorems that were proved were solved and they reinforced the proof much better. After the activities were done, the knowledge test was administered as a post-test in the fifth week.

**FINDINGS**

The responses of the students were evaluated over 100 points according to the Mathematical Knowledge Test Point Scale. The scores students got from the pre-test and post-test and the codes composed from their names and surnames were given in Table 2.

<table>
<thead>
<tr>
<th>Student</th>
<th>Pre.T</th>
<th>Post.T</th>
<th>Student</th>
<th>Pre.T</th>
<th>Post.T</th>
<th>Student</th>
<th>Pre.T</th>
<th>Post.T</th>
</tr>
</thead>
<tbody>
<tr>
<td>CK</td>
<td>0</td>
<td>40</td>
<td>KA</td>
<td>25</td>
<td>0</td>
<td>AB</td>
<td>40</td>
<td>57</td>
</tr>
<tr>
<td>MÇ1</td>
<td>30</td>
<td>87</td>
<td>ZÇ</td>
<td>0</td>
<td>60</td>
<td>BB</td>
<td>0</td>
<td>80</td>
</tr>
<tr>
<td>MK1</td>
<td>40</td>
<td>84</td>
<td>ŞÖ</td>
<td>5</td>
<td>20</td>
<td>HIG</td>
<td>25</td>
<td>54</td>
</tr>
<tr>
<td>BK</td>
<td>0</td>
<td>60</td>
<td>ST</td>
<td>5</td>
<td>70</td>
<td>SK1</td>
<td>25</td>
<td>27</td>
</tr>
<tr>
<td>ŞB</td>
<td>5</td>
<td>27</td>
<td>FK</td>
<td>5</td>
<td>24</td>
<td>SMF</td>
<td>10</td>
<td>30</td>
</tr>
<tr>
<td>RB</td>
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<td>47</td>
<td>UG</td>
<td>55</td>
<td>60</td>
<td>VA</td>
<td>30</td>
<td>27</td>
</tr>
<tr>
<td>MK2</td>
<td>5</td>
<td>47</td>
<td>MÇ2</td>
<td>5</td>
<td>50</td>
<td>SK2</td>
<td>50</td>
<td>77</td>
</tr>
<tr>
<td>SA</td>
<td>55</td>
<td>74</td>
<td>NBD</td>
<td>0</td>
<td>37</td>
<td>MZ</td>
<td>50</td>
<td>60</td>
</tr>
<tr>
<td>MT1</td>
<td>0</td>
<td>60</td>
<td>EK</td>
<td>0</td>
<td>67</td>
<td>AD</td>
<td>50</td>
<td>40</td>
</tr>
<tr>
<td>AK</td>
<td>0</td>
<td>34</td>
<td>ÜÖ</td>
<td>0</td>
<td>27</td>
<td>FBA</td>
<td>40</td>
<td>60</td>
</tr>
<tr>
<td>OE</td>
<td>5</td>
<td>20</td>
<td>MG</td>
<td>0</td>
<td>67</td>
<td>OM</td>
<td>40</td>
<td>94</td>
</tr>
<tr>
<td>AS</td>
<td>0</td>
<td>27</td>
<td>ÇŞ</td>
<td>5</td>
<td>51</td>
<td>RÇ</td>
<td>50</td>
<td>47</td>
</tr>
<tr>
<td>MT2</td>
<td>25</td>
<td>12</td>
<td>HSB</td>
<td>0</td>
<td>87</td>
<td>FD</td>
<td>15</td>
<td>38</td>
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<tr>
<td>BS</td>
<td>0</td>
<td>0</td>
<td>KŞ</td>
<td>5</td>
<td>47</td>
<td>SI</td>
<td>25</td>
<td>47</td>
</tr>
<tr>
<td>HA</td>
<td>0</td>
<td>34</td>
<td>YÇ</td>
<td>0</td>
<td>80</td>
<td>MD</td>
<td>30</td>
<td>47</td>
</tr>
</tbody>
</table>

Pre.T: Pre-Test  Post.T: Post-test

When Table 1 was examined, it was found that although students learned these theorems before, 15 participants could not answer the question and got zero in the pre-test. In the post-test, two students got zero. It was revealed that students got higher scores in the post-test. 39 students got higher grades than they did in the pre-test. Figure 1 examples from the participants’ test responses were given.
When the pre-test and post-test responses of the student coded as MÇ1 were examined, he got full points only with the first question in the pre-test and in the post-test, he could not get full score only from the seventh question. Therefore, MÇ1 got 30 points from the pre-test and 87 points from the post-test. When the Figure 2 was examined, anot-
her participant coded as OM got 40 points from the pre-test and 94 points from the post-test and she was the only student who got the highest score from the post-test among the other participants.

<table>
<thead>
<tr>
<th>Pre test</th>
<th>Post test</th>
</tr>
</thead>
</table>
| 3. \( f(1) + 2f(3) = x^2 + 2x \) fonksiyonunun ortalaması değeri hepsiyse:
| 4. \( x^2 + 2x = 0 \) denkleminin 0 ve 1 arasında bir kökün olup olmadığını gösterin. |
| 5. \( \lim_{{x \to 0}} \frac{\sin x}{x} = 1 \) olsun jura gösterin. |
| 6. \( \lim_{{x \to \infty}} \frac{1}{x} = 0 \) olsun jura gösterin. |
| Figure 2. The responses of a student coded as OM to the Mathematical Knowledge Test |
Figure 3. The responses of a student coded as YÇ to the Mathematical Knowledge Test

The student called YÇ got zero points because she could not answer any questions in the pre-test but in the post-test, she got 80 because of not answering only one question. In the post test, she answered four questions correctly. Moreover, dependent t-test analysis was performed to determine the significance of differences between the means and SPSS results were given in Table 3.
It was determined in Table 3 that there was a statistically significant difference between the scores students got from the pre-test and post-test in favour of the post-test scores (t(44)= -8.111, p<.05, r=.77). It is not only enough to look at the significance level to interpret a test result. At the same time, effect size must also be considered because while the result of the test can be significant, effect size can be low (Field, 2009). Effect size was calculated as:

\[ r = \frac{t}{\sqrt{t^2 + df}} = 0.77 \]

The effect size of the study carried out was r = 0.77 and when Cohen’s (1992) classification was considered, it was determined as a high value. In other words, it can be stated that students’ problem-solving skills increased thanks to the questions which were developed regarding the implementations of theorem proofs carried out with the help of mathematical models.

RESULTS, DISCUSSION AND SUGGESTIONS

Four questions in the pre-test and five questions in the post-test were asked in Mathematical Knowledge Test in order to discover the change in students’ academic achievement and the responses were evaluated over 100 points. The data obtained were analysed with SPSS software package. It was found that while the arithmetic means of the grades students got from the pre-test was 16.8, it was 48.5 in the post test. Dependent t-test was performed on the related scores to determine whether this difference was statistically significant or not. According to the dependent t-test results of the data, there was a significant difference in favour of post-test and also the effect size was high. These findings indicate that there was a big difference between the pre-test and post-test in terms of achievement. These two findings demonstrate that the instruction carried out has a considerable effect on students’ academic achievement. A similar result was found by Sandalcı (2013) and the study investigated the effect of using models in mathematics on 6th grade students’ algebra achievement. A statistically significant difference was found in the research and it meant that using a model promoted algebra achievement. It was found in many research studies that mathematical models and modelling had a positive effect on students’ academic achievement and also they increased their achievement and these findings show parallelism with this study (Çiltaş & Işık, 2012; Moslev & Jenaabadi, 2015).

Because deductive proofs actualized by the teacher are not understood by the students and they are copied and stored in short-term memory and they are not associated with daily life while proving theorems, as stated by Harrison (2001) thanks to models, visualization of complex abstract concepts, objects, and processes promote retention of knowledge in the mind. In the contrary case, the studies conducted revealed that instead of understanding, theorem proofs were memorized without making any effort to pass the exams, so it was observed that they were forgotten easily and students did not enjoy doing
By Polat and Akğün (2016), many people think that proof is the center of mathematics disciplines. Proof has been considered a fundamental part of mathematical practice science ancient times. It was introduced into the mathematics curriculum long ago and has been given an important status in teaching. During the new math movement, proof was stressed in other areas of mathematics as well. However, students have serious difficulties in learning proof. Reasoning is basis of mathematics. So the essence of mathematics is proving. Therefore, reasoning skills must be developed in students. In this framework, although students can understand mathematical proof at high school in mathematics and geometry courses, students have difficulty in understanding mathematical proof. Consequently, the students have lacked in proving skills when they come to university.

Constructing proofs for undergraduate studies is an inseparable part of mathematics education. However, students have difficulty in understanding this subject. Thus, doing proof activities with mathematical models course can be involved in undergraduate studies and students can understand proofs much better. In addition to this, students and teachers’ use of mathematical models can be developed. It is also suggested that sample activities in which models are used should be included in curricula at each level for students to understand models much better and use them in daily life. If teachers have more detailed knowledge about using models, they will be able to transfer them to their students much better. Thus, teachers can be offered in-service training and they can have more detailed knowledge about the use of models. But, time is one of the obstacles which prevent learning and doing proof with the help of mathematical models. Therefore, considering the time spared in curricula, lesson hours set for the concepts can be determined again. In this research, proofs of five theorems included in Analysis-I course were studied. Different proofs in different branches of mathematics can be studied. Moreover, studies can be carried out about exploring how students understand the concepts and subjects explained by using mathematical models not only at university level but also at much lower levels.

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